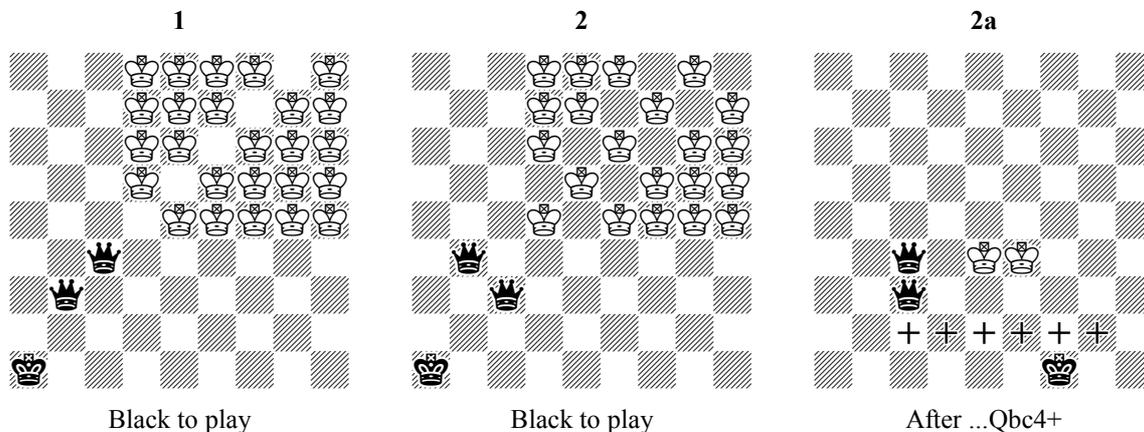


## Yet more on two queens against two moves

JDB reporting work by Noam Elkies, 21 September 2012

While I was working on what became *The Classified Encyclopedia of Chess Variants*, I came across a statement that in the game where White has fewer men than Black but has two moves at each turn, it was difficult for Black to mate without a formidable force – “Q+R+R was quoted by Twiss (1787)” (*The Encyclopedia of Chess Variants*, page 93, and the *Classified Encyclopedia*, page 26). It occurred to me to wonder why K+2Q would not also be sufficient, and in due course I found that they would (*Variant Chess* 53, page 145). But the method I used exploited the limited size of the 8x8 board, and in attempting to find how large the board could be I discovered a simple alternative method which worked on a board of any size (*Variant Chess* 54, page 163).

This alternative method had a measure of “just possible” elegance, the precise placing of the Black king in support of his queens appearing crucial, and I was therefore extremely surprised to receive a recent e-mail from Noam Elkies showing that the mate could be forced by the two Black queens acting alone. Noam has also been looking at positions where the lone king can avoid defeat and has sent me two positions where White can force a draw even though Black keeps two queens on the board, one of them being of considerable interest.

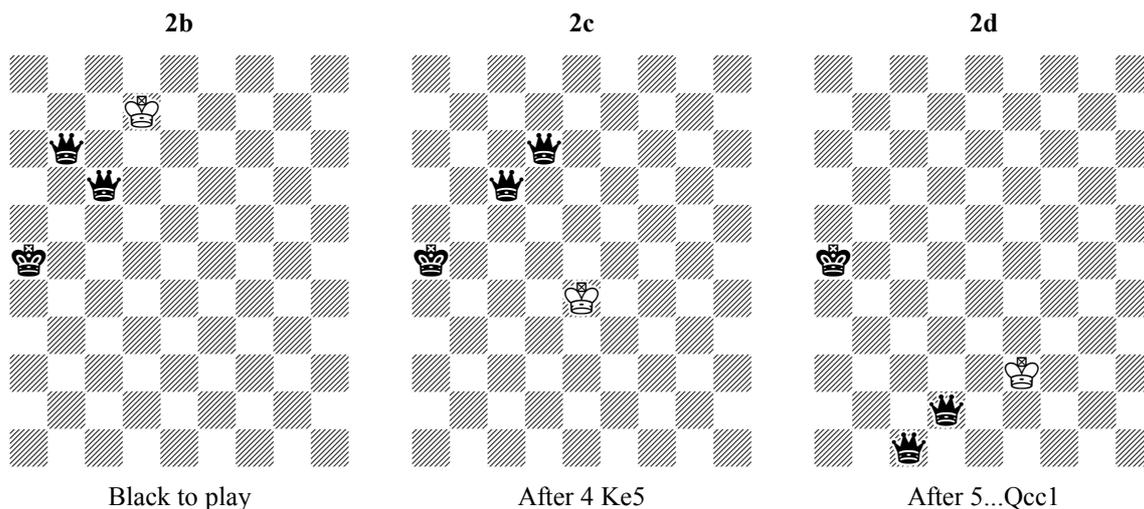


At the time of writing, all issues of *Variant Chess* can be downloaded free of charge from George Jelliss’s “Mayhematics” web site <[www.mayhematics.com](http://www.mayhematics.com)>, but even so it is convenient to start by recapitulating what appeared in issue 54. White’s first move is allowed to pass through check, so one queen is not sufficient to constrain his king and we need two queens covering adjacent ranks or files. We therefore assume that the queens are diagonally adjacent to each other, thus penning the White king into one of the four quarters of the board, and that the Black king is safely behind them. There are two cases, typified by **1** and **2**, and in each case Black’s objective is to move his queens steadily north-eastwards.

Case **1** is in fact very easy. If White is not on the fifth rank, Black can play ...Qbb5, gaining a rank upwards (we continue to denote the squares by a1 etc as on an ordinary diagram, even though the a-file and 1-rank are no longer edges of the board); if White is not on the d-file, Black can play ...Qbd3, gaining a file to the right. One or other of these moves is always possible.

A similar advance is possible in case **2** unless White is on d5. In this case, Black brings his king across to g1 and then plays ...Qbb3+, and White does best to play Ke4 or Kf4 since any other move will allow Black to play ...Qcc4 and set up case **1**. With White on e4 or f4, Black continues ...Qbc4+, giving **2a**, and we notice how the Black king and queens between them just cover all the accessible squares on the second rank. White must therefore retreat, and only Kd6 prevents an immediate ...Q4d4 gaining a file to the right. But after Kd6 Black can play ...Q3b3 once more obtaining case **1**, and the advance will duly follow. It’s a very simple five-move repetitive sequence, counting the queen advance at move 4 and a parallel king move at move 5, and it ends with the defender scrunched into the top right corner.

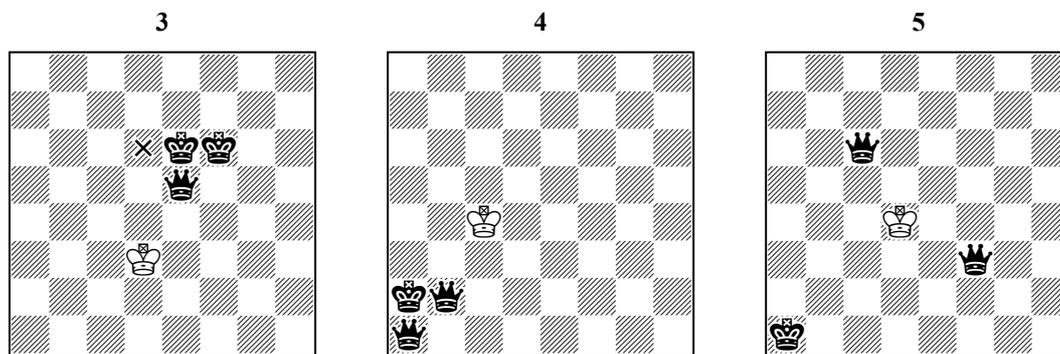
The move of the Black king to g1, so as to control f2/g2/h2 in diagram **2a**, appeared so obviously necessary that I was most surprised when Noam Elkies pointed out that it was nothing of the kind. Suppose that the Black king stays on the a-file, allowing White to play say Kf2; Black simply plays a queen to say d4, and he has gained a file to the right. True, the White king has come below the queens, but this doesn’t matter; Black now drives White downwards and rightwards instead of upwards and rightwards, and sooner or later White will have to yield another file. And we may be able to do better, since if White is significantly nearer to the top rank than to the bottom Black can play his queens below the White king and then resume driving him upwards. To avoid having to use ranks with negative numbers, let us renumber so that the queens are on b9 and c8 :



If the bottom edge is distant and the top edge relatively close, an improved line is now **1...Qbb8+ 2 Ke9/Kf9 Qbc9+ 3 Kf7** (if instead 3 Kd11 then 3...Q8b9+, which Noam points out to be quicker than the move 3...Q8b8 whose equivalent I gave in the analysis of 2 in *Variant Chess* 54) **Q9d9+ 4 Ke5** (see 2c) **Qd2 5 Kf3 Qcc1** (see 2d) with **6 Ke4 Qcc2+** and we have gained a file to the right at a cost of six ranks (compare the position after 1...Qbb8+) or **6 Kf3 Qce1** and we have gained two files at a cost of seven ranks (compare 2b itself). With this improvement, the length of the winning procedure on an  $n \times n$  board, although still greater than when the Black king was assisting, is again bounded by a linear function of  $n$ .

All this is valid only if the Black king is safely out of harm's way. What happens if he isn't?

K v K is a simple win for White, who plays to put his king two moves away from Black's and then drives him steadily back. K v K+Q may also be won, usually when the White king has a sequence of checks leading to mate or a fork, but Black draws if his queen is on a square away from the edge and his king is adjacent to her and not open to a check. In diagram 3, the Black king is safe on e6 or f6, but if he is on d6 White has the forking check Kc4+ driving him to the seventh rank and picking up the queen. And Black has two adjacent safe squares wherever the White king may be, so even if he is already on one of them he cannot be placed in zugzwang.



Either side to play : draw with bKe6/f6

Black to play : draw

Either side to play : draw!

To win or draw with K v K+2Q, White normally needs to play a sequence of checks ending in mate or capture of a queen. 4, from Noam, shows a simple draw by perpetual check. Black, in check, can only play **1...Kb1**, but he now threatens **2...Qaa3** shutting the White king off from the Black and eventually winning, and White can prevent this only by playing **2 Kd3+**. There follows **2...Ka2 3 Kc4+** repeating the initial position.

In 5, also sent to me by Noam, Black is not in check, and the play is much more interesting. Suppose first that Black is to play. To move a queen is to lose a queen or to allow a mate by Kc2/Kb3. Black can draw by **1...Qc1 2 Kxf3 Qb2**, but more challenging is **1...Ka2** threatening **2-7...Ka8** and **8...Qb7** winning (9 Kd6/Ke6 Qc3 etc, 9 Kc5/Kc4 Qf6, 9 K~ Qfc6). Hence **2 Kb4+**, and if **2...Kb1** then **3 Kd2+**. And if now **3...Ka1?** Not **4 Kb4**, when **4...Qc1** wins (5 Ka~/Kb~/Kd6 Qb2, 5 Kd4 Qfa3), nor **4 Kd2**, when **4...Qa3** wins, but **4 Kd4!** returning to the initial position.

And if White is to play in 5? Again drawn, but only if he plays the null move **1 Kd4!**

Are there any other drawn setups with K v K+2Q? As Noam rightly says, writing a program to find them would not be difficult, but at the moment there are more urgent things on my plate. Feel free to get in first.